Heterogeneous Fleet Routing with External Transporter

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Presentation outline

• Problem definition
• Literature
• Solutions
  – Mathematical model
  – Heuristics
• Results
• Conclusion
Problem hypothesis

- In this problem:
  Total Demand > Capacity of the internal fleet

- Heterogeneous limited fleet is insufficient to supply retailers

- External transporter is necessary

The question is...

- How to supply customers to minimize routing costs using heterogeneous internal fleet (fixed and variable costs) or external transporter (fixed cost)?
Problem definition

INDEX

- $n$: Number of retailers
- $m$: Number of vehicles in the internal fleet
- $i \in \{0, ..., n\}$: Index set of retailers (warehouse = 0)
- $j \in \{0, ..., n\}$: Index set of retailers (warehouse = 0)
- $k \in \{1, ..., m\}$: Index set of vehicles in the internal fleet

PARAMETERS

- $f_k$: Fixed cost of vehicle $k$
- $c_{ijk}$: Variable cost for vehicle $k$ traveling from retailer $i$ to retailer $j$
- $e_i$: Fixed cost to supply retailer $i$ with the external transporter
- $q_i$: Demand of retailer $i$
- $w_k$: Capacity of vehicle $k$

DECISION VARIABLES

- $X_{ijk} = \begin{cases} 
1 & \text{if the vehicle } k \text{ traveling from retailer } i \text{ to retailer } j \\
0 & \text{otherwise}
\end{cases}$
- $Y_{ik} = \begin{cases} 
1 & \text{if the demand of retailer } i \text{ is supplied by the vehicle } k \\
0 & \text{otherwise}
\end{cases}$
- $Z_i = \begin{cases} 
1 & \text{if the demand of retailer } i \text{ is supplied by the external transporter} \\
0 & \text{otherwise}
\end{cases}$
Problem definition

• Minimize:
  – Transportation Cost
  • Fixed and variable costs of the internal fleet
  • Fixed cost of the external transporter

• Subject to:
  – Supply all customers
  – Loading capacity of the vehicles
  – Availability of the vehicles
  – Subtours constraints

Literature

• Heterogeneous Vehicle Routing Problem with External Transporter
  – Chu (2005)

• Heterogeneous Vehicle Routing Problem (HVRP)
  – Osman & Salhi (1996)
  – Taillard (1999)
  – Gendreau et al. (1999)
  – Tarantilis et al. (2004)
### Resolution of the math. model

- **Example:**
  - **Datas:**
    - 15 retailers
    - 1 vehicle of type 1
    - 1 vehicle of type 2
    - 1 vehicle of type 3
  - **Formulation:**
    - 828 variables
    - 783 binary variables
  - **Résolution time:** 1.3 h
- **20 retailers:** hard to solve
- **Real life problems:** forget it!

### Heuristic overview

#### 1. CONSTRUCTION
  - Clarke & Wright (1964) with fleet limitation

#### 2. TRUCK ASSIGNMENT

#### 3. IMPROVEMENT

- **A 4-opt**
- **B Roads interchange**
  - i. Create a back-and-forth tour with an external customer (1-0 internal/external).
  - ii. Create an external tour with an internal customer (0-1 internal/external).
  - iii. Create a new tour with two external customers (2-0 internal/external).
  - iv. Switch internal customer with external customer (1-1 internal/external).
  - v. Osman 2-exchange
- **C Roads merge**
  - i. Merging 2 tours.
  - ii. Merging 3 tours.

#### 4. SOLUTION PERTURBATION
Solution perturbation process

- Predetermined % of perturbed node ($\alpha$)
- Predetermined perturbation cycle
  1. Nodes selection
  2. Move the nodes on other active roads or to the external transporter.
  3. Check the capacity of new routes
     - If the truck capacity is exceed, eject some nodes (selected randomly) until the capacity is respected

Instances of Chu (2005)

<table>
<thead>
<tr>
<th>Instance</th>
<th>Number of retailers</th>
<th>Optimal solution</th>
<th>Chu heuristic deviation with opt</th>
<th>Our heuristic deviation with opt</th>
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</thead>
<tbody>
<tr>
<td>Chu 1</td>
<td>5</td>
<td>387.50</td>
<td>387.50</td>
<td>387.50</td>
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<tr>
<td>Chu 2</td>
<td>10</td>
<td>586.00</td>
<td>631.00</td>
<td>586.00</td>
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<tr>
<td>Chu 3</td>
<td>15</td>
<td>823.50</td>
<td>900.00</td>
<td>823.50</td>
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<tr>
<td>Chu 4</td>
<td>22</td>
<td>1389.00</td>
<td>1,681.50</td>
<td>1,389.00</td>
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<tr>
<td>Chu 5</td>
<td>29</td>
<td>1,917.00</td>
<td>1,441.50</td>
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</table>

$\alpha$: 20%  Cycle: 20
New instances

<table>
<thead>
<tr>
<th>Instance</th>
<th>Number of retailers</th>
<th>Chu heuristic solution</th>
<th>Our heuristic solution</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12</td>
<td>668,00</td>
<td>666,00</td>
<td>0,30%</td>
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<tr>
<td>3</td>
<td>20</td>
<td>890,00</td>
<td>800,00</td>
<td>11,25%</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>1 048,00</td>
<td>976,00</td>
<td>7,38%</td>
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<tr>
<td>5</td>
<td>20</td>
<td>6 980,00</td>
<td>6 964,00</td>
<td>0,23%</td>
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<tr>
<td>6</td>
<td>20</td>
<td>1 241,00</td>
<td>977,00</td>
<td>27,02%</td>
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<tr>
<td>7</td>
<td>30</td>
<td>7 386,00</td>
<td>7 321,00</td>
<td>0,89%</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>7 441,00</td>
<td>2 310,00</td>
<td>5,67%</td>
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<tr>
<td>9</td>
<td>30</td>
<td>2 735,00</td>
<td>2 583,00</td>
<td>5,88%</td>
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<tr>
<td>10</td>
<td>30</td>
<td>2 203,00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>30</td>
<td>5 261,00</td>
<td>5 127,00</td>
<td>2,61%</td>
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<tr>
<td>12</td>
<td>30</td>
<td></td>
<td>3 696,00</td>
<td></td>
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<tr>
<td>13</td>
<td>50</td>
<td>2 720,00</td>
<td>2 454,00</td>
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<td>14</td>
<td>50</td>
<td>1 929,00</td>
<td>1 695,00</td>
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<td>2 570,00</td>
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<tr>
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<td>50</td>
<td>3 077,00</td>
<td>2 769,00</td>
<td>11,12%</td>
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<td>75</td>
<td>2 267,00</td>
<td>1 845,00</td>
<td>22,87%</td>
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<tr>
<td>18</td>
<td>75</td>
<td>3 114,00</td>
<td>2 776,00</td>
<td>12,18%</td>
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<tr>
<td>19</td>
<td>100</td>
<td>12 843,00</td>
<td>12 244,00</td>
<td>4,89%</td>
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<tr>
<td>20</td>
<td>100</td>
<td>4 828,00</td>
<td>4 313,00</td>
<td>11,94%</td>
</tr>
</tbody>
</table>

$\alpha$: 10%  Cycle: 2000  Average improvement 9,38%

Conclusion and future research

- Hard to deal with the limited and heterogeneous fleet.
- Availability of external transporters makes the problem much more difficult.
- We are able to solve this problem with better results than the actual literature.
- Future research: solve different versions of the problem.